Solving Rational Equations

By Cross Multiplying:

① $\dfrac{-1}{x-3} \diagup\!\!\!\!\times \dfrac{x-4}{x^2-27}$ $\left(\begin{array}{c}\text{Single} = \text{Single} \\ \text{fraction} \quad \text{fraction}\end{array}\right)$

$-1(x^2-27) = (x-3)(x-4)$

$-x^2 + 27 = x^2 - 7x + 12$

$+x^2 - 27 \quad\quad +x^2 \quad\quad\quad -27$

$$0 = 2x^2 - 7x - 15 \quad ac = -30$$

$$0 = 2x^2 - 10x + 3x - 15 \quad\quad b = -7$$

$$-10 \quad 3$$

$$0 = (2x)(x-5) + 3(x-5)$$

$$0 = (x-5)(2x+3)$$

$x - 5 = 0$ | $2x + 3 = 0$

$\boxed{x = 5}$ | $\boxed{x = \dfrac{-3}{2}} = -1.5$

$ck:$ ① $\dfrac{-1}{5-3} \overset{?}{=} \dfrac{5-4}{5^2-27}$ ② $\dfrac{-1}{-1.5-3} \overset{?}{=} \dfrac{-1.5-4}{(-1.5)^2-27}$

$\dfrac{-1}{2} = -\dfrac{1}{2} \checkmark$ $\dfrac{2}{9} = \dfrac{3}{9} \checkmark$

By Using LCD:

$LCD = 4x$

② $\left(\dfrac{4}{4}\right)\dfrac{5}{x} + \dfrac{7x}{4x} = -\dfrac{9}{x}\left(\dfrac{4}{4}\right)$

$4x \cdot \dfrac{20}{4x} + \dfrac{7x}{4x} = \dfrac{-36}{4x}$

* Multiply entire eq'n by the LCD to clear out the denominator

$$20 + 7x = -36$$
$$\underline{-20 \qquad\qquad -20}$$
$$7x = -56$$
$$\checkmark \boxed{x = -8}$$

CK: $\dfrac{5}{-8} + \dfrac{7}{4} \stackrel{?}{=} \dfrac{-9}{-8}$

$1.125 = 1.125 \checkmark$

2

② $x(x-5)\left[1 - \dfrac{8}{x-5} = \dfrac{3}{x}\right]$ $\dfrac{3}{x}\dfrac{(x-5)}{(x-5)}$ LCD = $x(x-5)$

$$x(x-5)\cdot 1 \qquad x-5 \qquad x$$

$$\boxed{\dfrac{x^2-5x}{x(x-5)} - \dfrac{8x}{x(x-5)} = \dfrac{3x-15}{x(x-5)}}$$

$$x^2 - 5x - 8x = 3x - 15$$

$$\underline{ -3x+15 \qquad -3x+15}$$

$$1x^2 - 16x + 15 = 0$$

$$\underline{(x-15)(x-1) = 0}$$

$$x = 15 \mid x = 1$$

$\underline{OK:}$ ①

$$1 - \dfrac{8}{15-5} \overset{?}{=} \dfrac{3}{15}$$

$$1 - \dfrac{8}{10} = \dfrac{1}{5}$$

$$\dfrac{10}{10} - \dfrac{8}{10}$$

$$\dfrac{2}{10} = \dfrac{1}{5}$$

$$\dfrac{1}{5} = \dfrac{1}{5} \checkmark$$

②

$$1 - \dfrac{8}{1-5} \overset{?}{=} \dfrac{3}{1}$$

$$1 - \dfrac{8}{-4} = 3$$

$$1 + 2 = 3$$

$$3 = 3 \checkmark$$

④ $\dfrac{9(x+2)}{(x-2)(x+2)} + \dfrac{6x(x-2)}{x+2(x-2)} = \dfrac{9x^2}{x^2-4}$ DOTS

$(x+2)(x-2)$ ← LCD

$(x+2)(x-2)$

$\left[\dfrac{9x+18}{(x+2)(x-2)} + \dfrac{6x^2-12x}{(x+2)(x-2)} = \dfrac{9x^2}{(x+2)(x-2)}\right]$

$9x + 18 + 6x^2 - 12x = 9x^2$

$6x^2 - 3x + 18 = 9x^2$
$-9x^2 \qquad\qquad\qquad -9x^2$
_____
$-3x^2 - 3x + 18 = 0$

GCF $\quad -3(x^2 + x - 6) = 0$

$-3(x+3)(x-2) = 0$

$-3 \ne 0 \quad \boxed{x = -3} \quad \boxed{x = 2}$

extraneous root
reject (makes original undefined)

CK: $\dfrac{9}{-3-2} + \dfrac{6(-3)}{-3+2} = \dfrac{9(-3)^2}{(-3)^2-4}$

$\dfrac{9}{-5} + \dfrac{-18}{-1} = \dfrac{81}{5}$

$16.2 = 16.2$ ✓